

## TEMA 12 – CÁLCULO DE PRIMITIVAS

**EJERCICIO :** Calcula las integrales:

- a)  $\int \frac{4x - 2 \cos x}{x^2 - \sin x} dx$       b)  $\int x e^{2x} dx$       c)  $\int \frac{x}{x^2 - 9} dx$       d)  $\int \frac{e^x}{1 + e^{2x}} dx$
- e)  $\int \frac{e^x}{1 + (e^x)^2} dx$       f)  $\int \frac{dx}{x^3 + 2x^2 + x}$       g)  $\int \frac{dx}{\sqrt[4]{3x+1}}$       h)  $\int (2x + 1) \cos x dx$
- i)  $\int \frac{2}{x^2 - 1} dx$       j)  $\int 2x(x - 1)^2 dx$       k)  $\int (x + 2) \operatorname{sen} x dx$       l)  $\int \frac{dx}{x^3 - 3x^2}$
- m)  $\int \frac{3x + \sqrt{x}}{x^2} dx$       n)  $\int (x + 3) e^{x+1} dx$       ñ)  $\int \frac{x}{x^2 + 2x - 8} dx$

*Solución:*

a)  $\int \frac{4x - 2 \cos x}{x^2 - \sin x} dx = \int \frac{2(2x - \cos x)}{x^2 - \sin x} dx = 2 \ln |x^2 - \sin x| + k$

b)  $\int x e^{2x} dx$ . Integramos por partes:  $\begin{cases} u = x \rightarrow du = dx \\ dv = e^{2x} dx \rightarrow v = \frac{1}{2} e^{2x} \end{cases}$

$$\int x e^{2x} dx = \frac{x}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx = \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} + k = \left(\frac{x}{2} - \frac{1}{4}\right) e^{2x} + k$$

c)  $\int \frac{x}{x^2 - 9} dx$ . Descomponemos en fracciones simples:

$$\frac{x}{x^2 - 9} = \frac{x}{(x - 3)(x + 3)} = \frac{A}{x - 3} + \frac{B}{x + 3} = \frac{A(x + 3) + B(x - 3)}{(x - 3)(x + 3)}$$

- Para  $x = 3 \rightarrow 3 = 6A \rightarrow A = \frac{1}{2}$

- Para  $x = -3 \rightarrow -3 = -6B \rightarrow B = \frac{1}{2}$

Por tanto:  $\int \frac{x}{x^2 - 9} dx = \int \left( \frac{\frac{1}{2}}{x - 3} + \frac{\frac{1}{2}}{x + 3} \right) dx = \frac{1}{2} \ln |x - 3| + \frac{1}{2} \ln |x + 3| + k$

d)  $\int \frac{e^x}{1 + e^{2x}} dx = \int \frac{e^x}{1 + (e^x)^2} dx = \operatorname{arctg}(e^x) + k$

e)  $\int (x + 2) \ln x dx$ . Integramos por partes:  $\begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = (x + 2) dx \rightarrow v = \frac{x^2}{2} + 2x \end{cases}$

$$\begin{aligned} \int (x + 2) \ln x dx &= \left(\frac{x^2}{2} + 2x\right) \ln x - \int \left(\frac{x^2}{2} + 2x\right) \frac{1}{x} dx = \left(\frac{x^2}{2} + 2x\right) \ln x - \int \left(\frac{x}{2} + 2\right) dx = \\ &= \left(\frac{x^2}{2} + 2x\right) \ln x - \frac{x^2}{4} - 2x + k \end{aligned}$$

f)  $\int \frac{dx}{x^3 + 2x^2 + x}$ . Descomponamos en fracciones simples:

$$\frac{1}{x^3 + 2x^2 + x} = \frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

- Para  $x=0 \rightarrow 1=A$
- Para  $x=-1 \rightarrow 1=-C \rightarrow C=-1$
- Para  $x=1 \rightarrow 1=4A+2B+C \rightarrow B=-1$

Por tanto:  $\int \frac{dx}{x^3 + 2x^2 + x} = \int \left( \frac{1}{x} + \frac{-1}{x+1} + \frac{-1}{(x+1)^2} \right) dx = \ln|x| - \ln|x+1| + \frac{1}{x+1} + k$

g)  $\int \frac{dx}{\sqrt[4]{3x+1}} = \int (3x+1)^{-1/4} dx = \frac{1}{3} \int 3 (3x+1)^{-1/4} dx = \frac{1}{3} \frac{(3x+1)^{3/4}}{3/4} + k = \frac{4 \sqrt[4]{(3x+1)^3}}{9} + k$

h)  $\int (2x+1)\cos x dx$ . Integramos por partes:  $\begin{cases} u = 2x+1 \rightarrow du = 2dx \\ dv = \cos x dx \rightarrow v = \sin x \end{cases}$

$$\int (2x+1)\cos x dx = (2x+1)\sin x - \int 2\sin x dx = (2x+1)\sin x + 2\cos x + k$$

i)  $\int \frac{2}{x^2-1} dx$ . Descomponemos en fracciones simples:

$$\frac{2}{x^2-1} = \frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

- Para  $x=1 \rightarrow 2=2A \rightarrow A=1$
- Para  $x=-1 \rightarrow 2=-2B \rightarrow B=-1$

Por tanto:  $\int \frac{2}{x^2-1} dx = \int \left( \frac{1}{x-1} + \frac{-1}{x+1} \right) dx = \ln|x-1| - \ln|x+1| + k$

j)  $\int 2x(x-1)^2 dx = \int 2x(x^2-2x+1) dx = \int (2x^3-4x^2+2x) dx = \frac{2x^4}{4} - \frac{4x^3}{3} + x^2 + k = \frac{x^4}{2} - \frac{4x^3}{3} + x^2 + k$

k)  $\int (x+2)\sin x dx$ . Integramos por partes:  $\begin{cases} u = x+2 \rightarrow du = dx \\ dv = \sin x dx \rightarrow v = -\cos x \end{cases}$

$$\int (x+2)\sin x dx = -(x+2)\cos x + \int \cos x dx = -(x+2)\cos x + \sin x + k$$

l)  $\int \frac{dx}{x^3-3x^2}$ . Descomponemos en fracciones simples:

$$\frac{1}{x^3-3x^2} = \frac{1}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} = \frac{Ax(x-3)+B(x-3)+Cx^2}{x^2(x-3)}$$

- Para  $x=0 \rightarrow 1=-3B \rightarrow B=-\frac{1}{3}$
- Para  $x=3 \rightarrow 1=9C \rightarrow C=\frac{1}{9}$
- Para  $x=1 \rightarrow 1=-2A-2B+C \rightarrow A=-\frac{1}{9}$

Por tanto:  $\int \frac{dx}{x^3-3x^2} = \int \frac{-1}{9x} dx + \int \frac{-1}{3x^2} dx + \int \frac{1}{9(x-3)} dx = \frac{-1}{9} \ln|x| + \frac{1}{3x} + \frac{1}{9} \ln|x-3| + k$

m)  $\int \frac{3x+\sqrt{x}}{x^2} dx = \int \left( \frac{3x}{x^2} + \frac{\sqrt{x}}{x^2} \right) dx = \int \left( \frac{3}{x} + x^{-3/2} \right) dx = 3\ln|x| + \frac{x^{-1/2}}{-1/2} + k = 3\ln|x| - \frac{2}{\sqrt{x}} + k$

n)  $\int (x+3)e^{x+1} dx$ . Integramos por partes:  $\begin{cases} u = x+3 \rightarrow du = dx \\ dv = e^{x+1} dx \rightarrow v = e^{x+1} \end{cases}$

$$\int (x+3)e^{x+1} dx = (x+3)e^{x+1} - \int e^{x+1} dx = (x+3)e^{x+1} - e^{x+1} + k = e^{x+1}(x+3-1) + k = (x+2)e^{x+1} + k$$

ñ)  $\int \frac{x}{x^2 + 2x - 8} dx$ . Descomponemos en fracciones simples:

$$\frac{x}{x^2 + 2x - 8} = \frac{x}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2} = \frac{A(x-2) + B(x+4)}{(x+4)(x-2)}$$

$$\text{- Para } x=2 \rightarrow 2 = 6B \rightarrow B = \frac{1}{3}$$

$$\text{- Para } x=-4 \rightarrow -4 = -6A \rightarrow A = \frac{2}{3}$$

$$\text{Por tanto: } \int \frac{x}{x^2 + 2x - 8} dx = \int \left( \frac{\frac{2}{3}}{x+4} + \frac{\frac{1}{3}}{x-2} \right) dx = \frac{2}{3} \ln|x+4| + \frac{1}{3} \ln|x-2| + k$$