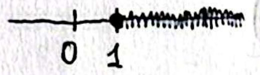
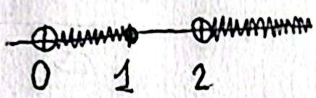
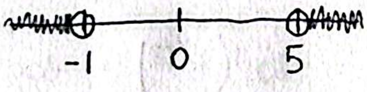
# Examen UD 1 y 2 [2021/2022]

**1**

| Recta Real | Intervalo | Formalmente |
|---|---|---|
| a)   | $[1, +\infty)$ | $\{x \in \mathbb{R} / x \geq 1\}$ |
| b)   | $(0, 1] \cup (2, +\infty)$ | $\{x \in \mathbb{R} / 0 < x \leq 1 \vee x > 2\}$ |

c) $|2x-4| > 6 \quad \begin{cases} 2x-4 > 6 \\ 2x-4 < -6 \end{cases} \Rightarrow \begin{cases} 2x > 10 \\ 2x < -2 \end{cases} \Rightarrow \begin{cases} x > 5 \\ x < -1 \end{cases}$




$(-\infty, -1) \cup (5, +\infty) \quad \{x \in \mathbb{R} / -1 > x > 5\}$

**2**

- $-\frac{58}{45} \Rightarrow$ Racional
- $\ln(e^{3000}) = 3000 \Rightarrow$ Natural
- $(\sqrt[3]{8})^{50} = 2^{50} \Rightarrow$ Natural
- $\frac{2\cancel{\pi}}{3\cancel{\pi}} = \frac{2}{3} \Rightarrow$ Racional
- $\frac{\pi}{3'14} \Rightarrow$ Irracional
- $\sqrt[5]{2^3} \Rightarrow$ Irracional
- $1'7898989\ldots \Rightarrow$ Racional
- $0 \Rightarrow$ Entero

3

a) $$\frac{2}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{2\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \boxed{\frac{2\sqrt[3]{2^2}}{2}}$$

b) $$\frac{3\sqrt{6}+2\sqrt{2}}{3\sqrt{3}+2} = \frac{(3\sqrt{6}+2\sqrt{2})\cdot(3\sqrt{3}-2)}{(3\sqrt{3}+2)\cdot(3\sqrt{3}-2)} =$$

$$\frac{9\sqrt{18}-\cancel{6\sqrt{6}}+\cancel{6\sqrt{6}}-4\sqrt{2}}{9\cdot 3-4} = \frac{9\sqrt{18}-4\sqrt{2}}{23} = \frac{9\cdot\sqrt{2\cdot 3^2}-4\sqrt{2}}{23} =$$

$$\frac{27\sqrt{2}-4\sqrt{2}}{23} = \frac{\cancel{23}\sqrt{2}}{\cancel{23}} = \boxed{\sqrt{2}}$$

4 $\log(A) = 0{'}5$ | $\log(B) = 3{'}3$

a) $$\log(\sqrt[3]{B}\cdot A) = \log(\sqrt[3]{B}) + \log(A) = \log(B^{1/3}) + \log(A) =$$

$$\frac{1}{3}\log(B) + \log(A) = \frac{3{'}3}{3} + 0{'}5 = 1{'}1 + 0{'}5 = \boxed{1{'}6}$$

b) $$\frac{1}{12345}\log\left(\frac{\sqrt[10]{A^4}}{(100B)^2}\right)^{12345} = \frac{\cancel{12345}}{\cancel{12345}}\cdot\log(\sqrt[10]{A^4}) - \log((100B)^2) =$$

$$\frac{4}{10}\cdot\log(A) - [2\log(100) + 2\log(B)] = \frac{2}{5}\log(A) - 4 - 2\cdot 3{'}3 =$$

$$\frac{2}{5}\cdot 0{'}5 - 4 - 6{'}6 = 0{'}2 - 4 - 6{'}6 = \boxed{-10{'}4}$$

## 5

a) $\ln(A) - \ln(A^2) = -1 \Rightarrow \ln\left(\frac{A}{A^2}\right) = \ln\left(\frac{1}{e}\right) \Rightarrow$

$\frac{A}{A^2} = \frac{1}{e} \Rightarrow A^{-1} = e^{-1} \Rightarrow \boxed{A = e}$

b) $\log_A(2^{12}) + \log_A(0'25) = 10 \Rightarrow \log_A(2^{12} \cdot 0'25) = 10 \Rightarrow$

$\log_A(2^{12} \cdot 2^{-2}) = \log_A(A^{10}) \Rightarrow A^{10} = 2^{10} \Rightarrow \boxed{A = 2}$

## 6

**DATOS**

$a_1 = 10$

$r = \frac{4}{5}$

**RESOLUCIÓN**

$a_n = 10 \cdot \left(\frac{4}{5}\right)^{n-1}$

$a_3 = 10 \cdot \left(\frac{4}{5}\right)^2 \Rightarrow a_3 = \frac{160}{25} = \boxed{\frac{32}{5}}$

$\begin{cases} a_1 = 10 \\ a_n = a_{n-1} \cdot \frac{4}{5} \end{cases}$

$S_\infty = \frac{a_1}{1-r} \Rightarrow S_\infty = \frac{10}{1-\frac{4}{5}} \Rightarrow$

$S_\infty = \frac{10}{1/5} = \frac{10}{1} : \frac{1}{5} = \boxed{50}$

**CONCLUSIÓN**

$a_3 = \frac{32}{5}$ ; $a_n = 10 \cdot \left(\frac{4}{5}\right)^{n-1}$ ; $S_\infty = 50$ ; $\begin{cases} a_1 = 10 \\ a_n = a_{n-1} \cdot \frac{4}{5} \end{cases}$

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a) $\lim \left( \dfrac{n^2+3n+1}{2n^2+n} \right) = \boxed{\dfrac{1}{2}}$

b) $\lim \left( \dfrac{5+3n}{n^2+2} \right) = \boxed{0}$

c) $\lim \left( (-1)^n + n^2 \right) = \boxed{\infty}$

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$\{2, 5, 11, 20, 32, \dots\}$

$+3 \quad +6 \quad +9 \quad +12$

$+3 \quad +3 \quad +3$

Orden 2 $\Rightarrow a_n = An^2 + Bn + C$

* Para este ejercicio, tomaremos o etiquetaremos el primer término como $a_0$

$\Rightarrow a_0 = a\cdot 0^2 + b\cdot 0 + c \Rightarrow a_0 = C \Rightarrow C = 2$

$a_1 = a + b + 2$

$a_2 = 4a + 2b + 2$

$\Rightarrow \begin{cases} a+b+2=5 \\ 4a+2b+2=11 \end{cases} \Rightarrow \begin{array}{r} \cancel{4a} + 4b = 12 \\ -\ \cancel{4a} + 2b = 9 \\ \hline 2b = 3 \end{array} \Rightarrow \boxed{b = \frac{3}{2}}$

$a + \frac{3}{2} + 2 = 5 \Rightarrow \boxed{a = \frac{3}{2}}$

Conclusión

$$\boxed{a_n = \frac{3}{2}n^2 + \frac{3}{2}n + 2}$$

Comprobación

$a_1 = \frac{3}{2} + \frac{3}{2} + 2 = 5$ ✓✓

$a_2 = 6 + 3 + 2 = 11$ ✓✓

$a_3 = \frac{27}{2} + \frac{9}{2} + 2 = \frac{40}{2} = 20$ ✓✓