

①

$$a) \log x + \log(x+3) = 2 \log(x+1)$$

$$\log(x(x+3)) = \log(x+1)^2$$

$$x^2 + 3x = x^2 + 2x + 1$$

$$\boxed{x = 1}$$

$$b) \frac{x-3}{x^2-6x+9} - \frac{2x}{x-3} + 3 = 0 \rightarrow x^2 - 6x + 9 \rightarrow x = \frac{6 \pm \sqrt{36-36}}{2} = \frac{6 \pm 0}{2} = \boxed{3}$$

$$\frac{1 \cdot \cancel{(x-3)}}{\cancel{(x-3)} \cdot \cancel{(x-3)}} - \frac{2x}{\cancel{(x-3)}} + \frac{3(x-3)}{\cancel{(x-3)}} = 0$$

$$1 - 2x + 3x - 9 = 0$$

$$\boxed{x = 8}$$

$$c) \sqrt{3-2x} + \sqrt{1-x} = 5$$

$$(\sqrt{3-2x})^2 = (5 - \sqrt{1-x})^2$$

$$3-2x = 25 - 10\sqrt{1-x} + 1-x$$

$$(10\sqrt{1-x})^2 = (x+23)^2$$

$$100(1-x) = x^2 + 46x + 529$$

$$x^2 + 146x + 429 = 0$$

$$x = \frac{-146 \pm \sqrt{(146)^2 - 4 \cdot 1 \cdot 429}}{2} = \frac{-146 \pm 140}{2} = \begin{cases} \frac{-6}{2} = \boxed{-3} \\ \frac{-286}{2} = \boxed{-143} \end{cases}$$

Comprobar

$$x = -143$$

$$\sqrt{3+286} + \sqrt{1+143} = 5$$

$$17 + 12 \neq 5$$

$$x = -3$$

$$\sqrt{3+6} + \sqrt{1+3} = 5$$

$$3 + 2 = 5 \checkmark$$

Solución

$$x = -3$$

②

$$a) \begin{cases} \log(x^2+y) - \log(x-2y) = 1 \rightarrow \log \frac{x^2+y}{x-2y} = \log 10 \rightarrow x^2+y = 10x-20y \rightarrow x^2-10x+21y=0 \\ 5^{x+1} = 25^{y+1} \rightarrow 5^{x+1} = (5^2)^{y+1} \rightarrow 5^{x+1} = 5^{2y+2} \rightarrow x+1 = 2y+2 \rightarrow y = \frac{x-1}{2} \end{cases}$$

$$x^2 - 10x + 21\left(\frac{x-1}{2}\right) = 0$$

$$\frac{2x^2}{2} - \frac{20x}{2} + \frac{21x-21}{2} = 0$$

$$2x^2 + x - 21 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-21)}}{4} = \frac{-1 \pm 13}{4} = \begin{cases} \frac{12}{4} = 3 \\ \frac{-14}{4} = -\frac{7}{2} \end{cases}$$

Si $x = 3$

$$y = \frac{3-1}{2} = 1$$

Si $x = -\frac{7}{2}$

$$y = \frac{-\frac{7}{2} - 1}{2} = \frac{-\frac{7}{2} - \frac{2}{2}}{2} = -\frac{9}{4}$$

Solución
$x = 3, y = 1$
$x = -\frac{7}{2}, y = -\frac{9}{4}$

$$b) \begin{cases} 2x+y-z=0 \\ x-y+2z=5 \\ x+y+z=3 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & -1 & 2 & 5 \\ 1 & 1 & 1 & 3 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 3 \end{array} \right) \rightarrow$$

$$\begin{array}{l} f_2 = f_2 - 2f_1 \\ f_3 = f_3 - f_1 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & 3 & -5 & -10 \\ 0 & 2 & -1 & -2 \end{array} \right) \xrightarrow{f_3 = 3f_3 - 2f_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & 3 & -5 & -10 \\ 0 & 0 & 7 & 14 \end{array} \right) \Rightarrow$$

$$\Rightarrow \begin{cases} x-y+2z=5 \rightarrow x-0+4=5 \rightarrow x=1 \\ 3y-5z=-10 \rightarrow 3y-10=-10 \rightarrow y=0 \\ 7z=14 \rightarrow z=2 \end{cases}$$

Solución
$x=1, y=0, z=2$

③

DATOS

a) Pasos de Ben 3 = Pasos de Z en 2 + 30 pasos.

no escalones = x

Operaciones

$$\left\{ \begin{array}{l} \frac{x}{3} + 30 = \frac{x}{2} \end{array} \right.$$

$$\frac{x+90}{3} = \frac{x}{2}$$

$$2x + 180 = 3x$$

$$\boxed{x = 180}$$

Resultado

El faro tiene 180 escalones

b) DATOS

456 caballos entre sus hijos.

2 chicos.

x chicas

{ 2 chicos sin caballos
chicas 19 caballos más cada una.

Operaciones

$$\left\{ \begin{array}{l} \frac{456}{2+x} = y \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{456}{x} = y + 19 \rightarrow y = \frac{456 - 19x}{x} \end{array} \right.$$

$$\frac{456 - 19x}{x} = \frac{456}{2+x}$$

$$(2+x)(456-19x) = 456x$$

$$912 + 418x - 19x^2 = 456x$$

$$19x^2 + 38x - 912 = 0$$

$$x = \frac{-38 \pm \sqrt{(38)^2 - 4 \cdot 19 \cdot (-912)}}{38} = \frac{-38 \pm 266}{38}$$

$\boxed{6}$

$\boxed{-8}$

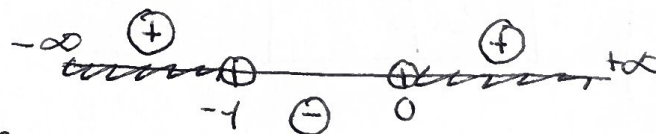
Resultado

El tío tiene 6 hijas

④

a) $\frac{x(x+1)}{5} > 0 \rightarrow x(x+1) > 0$

↓
las raíces son -1 y 0.



$\rightarrow x = -2 \rightarrow -2(-1) > 0 \oplus$

$\rightarrow x = -0.5 \rightarrow -0.5(0.5) > 0 \ominus$

$\rightarrow x = 1 \rightarrow 1(2) > 0 \oplus$

Solución

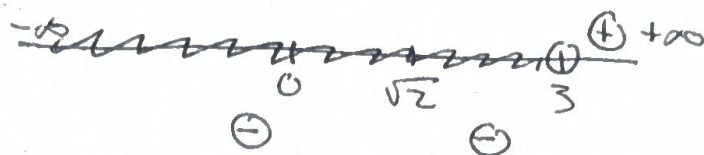
$$x \in (-\infty, -1) \cup (0, +\infty)$$

④

$$b) \frac{x^2+2}{x-3} \leq 0 \rightarrow$$

$$x^2+2=0 \rightarrow x=\sqrt{2}$$

$$x-3=0 \rightarrow x=3.$$



Solución
 $x \in (-\infty, 3)$

$$\rightarrow x=1 \rightarrow \frac{1+2}{1-3} \leq 0 \quad (-)$$

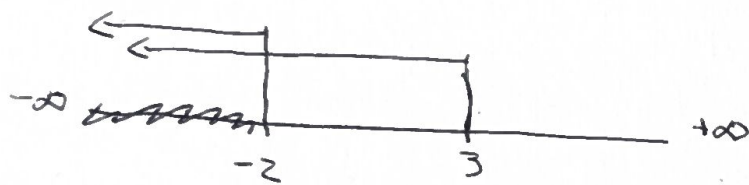
$$\rightarrow x=2 \rightarrow \frac{4+2}{2-3} \leq 0 \quad (-)$$

$$\rightarrow x=4 \rightarrow \frac{16+2}{4-3} \leq 0 \quad (+)$$

⑤

$$a) \begin{cases} 3x+8 \leq x+14 \rightarrow 2x \leq 6 \rightarrow x \leq 3. \\ 2x < \frac{3}{2}x - 1 \rightarrow \frac{1}{2}x < -1 \rightarrow x < -2 \end{cases}$$

Solución
 $x \in (-\infty, -2)$



$$b) \begin{cases} 2x-y+6 > 0 \rightarrow y < 2x+6 \text{ ec. 1} \\ -4x+2y > 2 \rightarrow y > 1+2x \text{ ec. 2.} \end{cases}$$

ec. 1.

x	y
1	8
-1	4

ec. 2

x	y
0	1
2	5

