Examen Tema 3 Álgebra 2021/2022:

1

a) $\log x + \log(x+3) = 2\log(x+1) \Rightarrow$

$\log[x(x+3)] = \log(x+1)^2 \Rightarrow \log(x^2+3x) = \log(x+1)^2 \Rightarrow$

$x^2+3x = x^2+1+2x \Rightarrow x^2+3x-x^2-1-2x=0 \Rightarrow x-1=0$

$\Rightarrow \boxed{x=1}$

b) $\dfrac{x-3}{\underbrace{x^2-6x+9}} - \dfrac{2x}{x-3} + 3 = 0$

factorizamos este denominador.

$x = \dfrac{6 \pm \sqrt{(-6)^2-4\cdot1\cdot9}}{2\cdot1} = \dfrac{6\pm\sqrt{36-36}}{2} = \dfrac{6\pm0}{2} = \boxed{x_1 = 3}, \boxed{x_2 = 3}$

Factor $(x-3)^2 \rightarrow$ Raíz

$\Rightarrow \dfrac{x-3}{(x-3)^2} - \dfrac{2x}{x-3} + 3 = 0 \Rightarrow \dfrac{1}{\cancel{x-3}} - \dfrac{2x}{\cancel{x-3}} + \dfrac{3(x-3)}{\cancel{x-3}} = \dfrac{0}{\cancel{x-3}} \Rightarrow$

$\Rightarrow 1-2x+3x-9=0 \Rightarrow \boxed{x=8}$

c) $\sqrt{3-2x} + \sqrt{1-x} = 5 \Rightarrow (\sqrt{3-2x})^2 = (5-\sqrt{1-x})^2 \Rightarrow$

$\Rightarrow 3-2x = 25+1-x-10\sqrt{1-x} \Rightarrow -10\sqrt{1-x} = 3-2x-25-1+x \Rightarrow$

$\Rightarrow \sqrt{1-x} = \dfrac{-x-23}{-10} \Rightarrow (\sqrt{1-x})^2 = \left(\dfrac{x+23}{10}\right)^2 \Rightarrow 1-x = \dfrac{x^2+529+46x}{100} \Rightarrow$

$\Rightarrow 100-100x = x^2+529+46x \Rightarrow x^2+529+46x+100x-100=0$

$\Rightarrow x^2+146x+429=0 \Rightarrow x = \dfrac{-146\pm\sqrt{146^2-4\cdot1\cdot429}}{2\cdot1} \Rightarrow$

$\Rightarrow x = \dfrac{-146\pm\sqrt{21316-1716}}{2} \Rightarrow x = \dfrac{-146\pm\sqrt{19600}}{2} \Rightarrow x = \dfrac{-146\pm140}{2}$

$\Rightarrow x = \boxed{x_1=-3} \Rightarrow \sqrt{9}+\sqrt{4} = 5$ V. Sí.

$\boxed{x_2 = -143} \Rightarrow \sqrt{289}+\sqrt{144} \neq 5 \rightarrow$ NO.

Única solución válida: $\boxed{x=-3}$

2

a) $\begin{cases} \log(x^2+y) - \log(x-2y) = 1 \\ 5^{x+1} = 25^{y+1} \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} \log\left(\frac{x^2+y}{x-2y}\right) = \log 10 \\ 5^{x+1} = 5^{2y+2} \end{cases} \Rightarrow \begin{cases} x^2+y = 10x-20y \\ x+1 = 2y+2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} \boxed{x^2-10x+21y=0} \ (1) \\ x-2y=1 \Rightarrow \boxed{x=1+2y} \ (2) \end{cases}$

Sustituyo (1) en (2)

$(1+2y)^2 - 10\cdot(1+2y) + 21y = 0 \Rightarrow 1+4y^2+4y-10-20y+21y=0$

$\Rightarrow 4y^2+5y-9=0 \Rightarrow y = \frac{-5\pm\sqrt{5^2-4\cdot4\cdot(-9)}}{2\cdot4} = \frac{-5\pm\sqrt{25+144}}{8} \Rightarrow$

$\Rightarrow y = \frac{-5\pm13}{8}$ $\nearrow \boxed{y_1=1} \Rightarrow x_1 = 1+2\cdot1 \Rightarrow \boxed{x_1=3}$

$\searrow \boxed{y_2=-9/4} \Rightarrow \boxed{x_2 = 1+2\cdot\left(\frac{-9}{4}\right) = 1-\frac{18}{4} = \frac{4-18}{4} = \frac{-7}{2}}$

Compruebo las soluciones:

- $x=3;\ y=1 \Rightarrow \log 10 - \cancel{\log 1}^{\,0} = 1 \Rightarrow \log 10 = 1$ ✓.
  $\Rightarrow 5^4 = 5^{2(1+1)} \Rightarrow 5^4 = 5^4$ ✓.
  } Sí es sol.

- $x=\frac{-7}{2};\ y=\frac{-9}{4} \Rightarrow \log\left[\left(\frac{-7}{2}\right)^2+\left(\frac{-9}{4}\right)\right] - \log\left[\left(-\frac{7}{2}-2\cdot\left(\frac{-9}{4}\right)\right)\right] = 1$
  $\Rightarrow \log 10 - \underbrace{\cancel{\log 1}}_{0} = 1 \Rightarrow \log 10 = 1$ ✓.
  $5^{-5/2} = 5^{2(-5/4)} \Rightarrow 5^{-5/2} = 5^{-5/2}$ ✓.
  } Sí es solución.

Conclusión: los soluciones son: $\boxed{\begin{array}{l} x_1=3;\ y_1=1 \\ x_2=\frac{-7}{2};\ y_2=\frac{-9}{4} \end{array}}$

2

b) $\begin{cases} 2x+y-z=0 \\ x-y+2z=5 \\ x+y+z=3 \end{cases} \Longrightarrow \left(\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 5 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 3 \end{array}\right) \xrightarrow[F_3=F_3-F_1]{F_2=F_2-2F_1} \left(\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 5 \\ 0 & 3 & -5 & -10 \\ 0 & 2 & -1 & -2 \end{array}\right)$

$\xrightarrow{F_2=F_2\cdot\left(\frac{1}{3}\right)} \left(\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 5 \\ 0 & 1 & -5/3 & -10/3 \\ 0 & 2 & -1 & -2 \end{array}\right) \xrightarrow{F_3=F_3-2F_2} \left(\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 5 \\ 0 & 1 & -5/3 & -10/3 \\ 0 & 0 & 7/3 & 14/3 \end{array}\right) \Longrightarrow$

$\Longrightarrow \begin{cases} x-y+2z=5 \\ y-\frac{5}{3}z=-\frac{10}{3} \Rightarrow y-\frac{5}{3}\cdot 2=\frac{-10}{3} \Rightarrow y=-\frac{10}{3}=\frac{-10}{3} \Rightarrow \boxed{y=0} \\ \frac{7}{3}z=\frac{14}{3} \Rightarrow \boxed{z=\frac{14}{3}:\frac{7}{3}=\frac{42}{21}=2} \end{cases}$

$x-0+2\cdot 2=5 \Rightarrow x-0+4=5 \Rightarrow \boxed{x=1}$

Solución:
$x=1$
$y=0$
$z=2$

3 a) DATOS:

Nº de escalones → x

"Si sube de tres en tres" → $\frac{x}{3}$

"tiene quedar 30 pasos menos que si los sube de 2 en 2" → $\frac{x}{3}=\frac{x}{2}-30$

OPERACIÓN:

$\frac{x}{3}=\frac{x}{2}-30 \Rightarrow \frac{2x}{\cancel{6}}=\frac{3x-180}{\cancel{6}} \Rightarrow -x=-180$

$\Rightarrow \boxed{x=180}$

Solución: El faro tiene 180 escalones.

b) DATOS: 456 caballos < Hijos (2), Hijas (x)

y (nº de caballos que le toca a cada uno).

Situación 1

Antes del enfado:

$456=2\cdot y+x\cdot y$

Situación 2

Después del enfado:

$456=x\cdot y+19\cdot x$

$$\begin{cases} \boxed{xy + 2y = 456} \ (1) \\ xy + 19x = 456 \Rightarrow \boxed{y = \dfrac{456 - 19x}{x}} \ (2) \end{cases}$$

Sustituyo (1) en (2):

$$\cancel{x} \cdot \left(\frac{456 - 19x}{\cancel{x}}\right) + 2 \cdot \left(\frac{456 - 19x}{x}\right) = 456 \Rightarrow 456 - 19x + \frac{912 - 38x}{x} = 456$$

$$\Rightarrow 456x - 19x^2 + 912 - 38x = 456x \Rightarrow -19x^2 - 38x + 912$$

$$\Rightarrow -x^2 - 2x + 48 \Rightarrow x = \frac{2 \pm \sqrt{2^2 - 4(-1)48}}{2 \cdot (-1)} \Rightarrow x = \frac{2 \pm \sqrt{196}}{-2} \Rightarrow$$

$$\Rightarrow x = \frac{2 \pm 14}{-2} \begin{cases} \boxed{\cancel{x_1 = -8}} \rightarrow \text{descartamos sol} < 0. \\ \boxed{x_2 = 6} \end{cases}$$

$\Downarrow$

SOL: Mi tío tiene 6 hijas.

4)

a) $\dfrac{x(x+1)}{5} > 0 \Rightarrow x(x+1) > 0 = \begin{cases} \boxed{x = 0} \\ x + 1 = 0 \Rightarrow \boxed{x = -1} \end{cases}$

Compruebo $x = -2$:

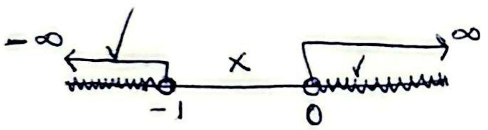
$$\frac{-2(-2+1)}{5} = \frac{+2}{+5} > 0 \Rightarrow \text{Sí}.$$

Compruebo $x = -0'5$:

$$\frac{-0'5(-0'5+1)}{5} = \frac{-}{+} = - \Rightarrow \text{No}.$$

Compruebo $x = 1$:

$$\frac{1(1+1)}{5} = \frac{2}{5} > 0 \Rightarrow \text{Sí}.$$



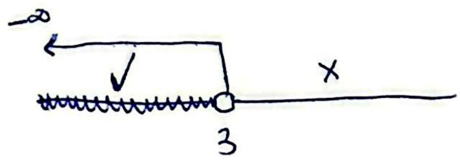
$\boxed{\begin{array}{l} \text{SOL: } (-\infty, -1) \cup (0, \infty) \\ \{x \in \mathbb{R} / x < -1 \vee x > 0\} \end{array}}$

b) $\dfrac{x^2+2}{x-3} \leq 0$

- Numerador: $x^2 + 2 = 0 \Rightarrow x^2 = -2$
$\Rightarrow x = \sqrt{-2} \Rightarrow \nexists$ sol real.

- Denominador: $x - 3 = 0 \Rightarrow \boxed{x = 3}$

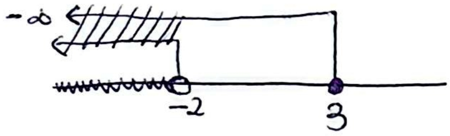
	$(-\infty, 3)$	$(3, \infty)$
x^2+2	+	+
$x-3$	−	+
	−	+
	✓	X



$\boxed{\begin{array}{l} \text{SOL: } (-\infty, 3) \\ \{x \in \mathbb{R} / x < 3\} \end{array}}$

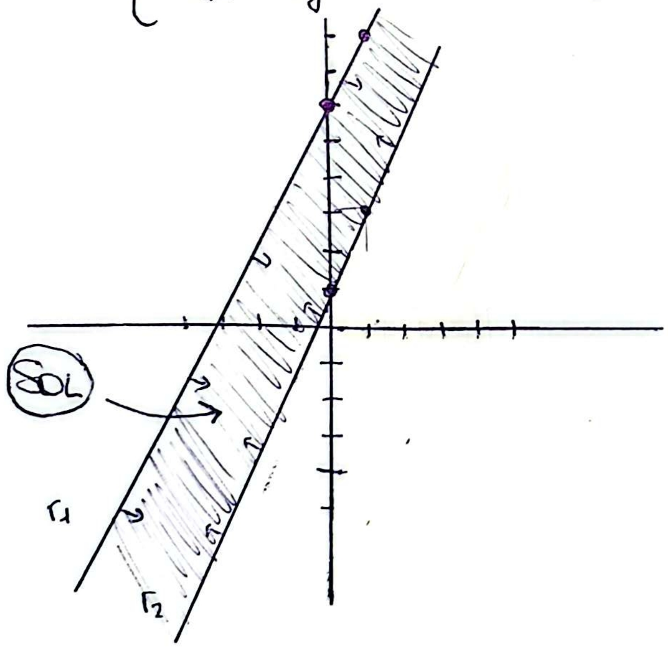
5

a) $\begin{cases} 3x+8 \leq x+14 \\ 2x < \frac{3}{2}x - 1 \end{cases} \Rightarrow \begin{cases} 2x \leq 6 \Rightarrow \boxed{x \leq 3} \\ \frac{1}{2}x < -1 \Rightarrow \boxed{x < -2} \end{cases}$



$\boxed{\underline{\text{SOL}}: \ (-\infty, -2) \quad \{x \in \mathbb{R} \,/\, x < -2\}}$

b) $\begin{cases} 2x - y + 6 > 0 \\ -4x + 2y > 2 \end{cases} \Rightarrow \begin{cases} 2x - y > -6 \Rightarrow -y > -6 - 2x \Rightarrow \boxed{y < 2x+6} \\ -4x + 2y > 2 \Rightarrow 2y > 2 + 4x \Rightarrow \boxed{y > 1 + 2x} \end{cases}$



$r_1: y = 2x + 6$

x	y
0	+6
1	8

$r_2: y = 1 + 2x$

x	y
0	1
1	3