EXAMEN UNIDAD 3 (PARTE I)

2025/2026

Ejercicio 1:

a) $\frac{\cancel{x+1}}{\cancel{x+1}} + \frac{x-1}{x-2} = \frac{2x+1}{x+1}$

$1 + \frac{x-1}{x-2} = \frac{2x+1}{x+1} \Rightarrow \frac{(x-2)(x+1)+(x+1)(x-1)}{\cancel{(x-2)(x+1)}} = \frac{(2x+1)(x-2)}{\cancel{(x-2)(x+1)}}$

$\Rightarrow x^2 + x - 2x - 2 + x^2 - 1 = 2x^2 - 4x + x - 2 \Rightarrow$

$\Rightarrow \cancel{x^2} + x - 2x - 2 + \cancel{x^2} - 1 - \cancel{2x^2} + 4x - x + 2 = 0 \Rightarrow$

$\Rightarrow 2x - 1 = 0 \Rightarrow \boxed{x = 1/2}$

b) $\left(\sqrt{x + \sqrt{2x-1}}\right)^2 = 3^2 \Rightarrow x + \sqrt{2x-1} = 9 \Rightarrow$

$\Rightarrow^{(*)} \left(\sqrt{2x-1}\right)^2 = (9-x)^2 \,(*) \Rightarrow 2x - 1 = 81 + x^2 - 18x \Rightarrow$

$\Rightarrow 81 + x^2 - 18x + 1 - 2x = 0 \Rightarrow x^2 - 20x + 82 = 0$

$\Rightarrow x = \frac{20 \pm \sqrt{(-20)^2 - 4 \cdot 1 \cdot 82}}{2 \cdot 1} \Rightarrow x = \frac{20 \pm \sqrt{72}}{2}$

$\rightarrow \frac{20 + \sqrt{2^3 \cdot 3^2}}{2} = \boxed{\frac{20 + 6\sqrt{2}}{2}}$

$\rightarrow \boxed{\frac{20 - 6\sqrt{2}}{2}}$

72	2
36	2
18	2
9	3
3	3
1	

$2^3 \cdot 3^2$

Soluciones

$\Rightarrow x_1 = 10 + 3\sqrt{2}$

$\Rightarrow x_2 = 10 - 3\sqrt{2}$

$\rightarrow$ pero ... $2x - 1 \geq 0 \Rightarrow \boxed{x \geq 1/2}$

$\underbrace{9 - x \geq 0}_{\text{porque } (*)} \Rightarrow \boxed{x \leq 9}$

La solución de una raíz daba $9 - x$, y eso siempre tiene que ser > 0.

Entonces...

$\rightarrow$ la única solución que cumple las dos condiciones es...

$\boxed{x = 10 - 3\sqrt{2}} \leftarrow$ Solución

c) $2^{2x}+6=5\cdot 2^{x} \Rightarrow 2^{2x}-5\cdot 2^{x}+6=0 \Rightarrow C.V \rightarrow \boxed{t=2^{x}}$

$\Rightarrow t^2-5t+6=0 \Rightarrow t=\frac{5\pm\sqrt{(-5)^2-4\cdot 1\cdot 6}}{2\cdot 1}=\frac{5\pm\sqrt{1}}{2}$

$\Rightarrow t = \begin{cases} t_1=\frac{6}{2}=3 \rightarrow 2^{x}=3 \rightarrow \log_2 2^{x}=\log_2 3 \Rightarrow \boxed{x=\log_2 3} \\ t_2=\frac{4}{2}=2 \rightarrow 2^{x}=2 \rightarrow \boxed{x=1} \end{cases}$

d) $\log_x(x-1+\ln(x))=1$

$\cancel{\log_x}(x-1+\ln(x))=\cancel{\log_x}x$

$x-1+\ln(x)=x \Rightarrow \ln(x)=1 \Rightarrow \boxed{x=e}$

Ejercicio 2:

$\begin{cases} \log(x+y)+\log(x-y)=\log(33) & (1) \\ \log_5(e^{x})+\log_5(e^{y})=\log_5(e^{11}) & (2) \end{cases}$

$\begin{cases} (1)\ \log(x^2-y^2)=\log 33 \Rightarrow \boxed{x^2-y^2=33} \\ (2)\ \log_5(e^{x+y})=\log_5(e^{11}) \Rightarrow e^{x+y}=e^{11} \Rightarrow \boxed{x+y=11} \end{cases}$

$\begin{cases} x^2-y^2=33 \\ x+y=11 \Rightarrow \boxed{x=11-y} \end{cases} \rightarrow (11-y)^2-y^2=33$

$121+\cancel{y^2}-22y-\cancel{y^2}=33$

$-22y=-88 \Rightarrow \boxed{y=4}$

$x+y=11 \Rightarrow \boxed{x=7}$

$\boxed{\text{SOLUCIÓN: } x=7,\ y=4}$

Escaneado con CamScanner

Ejercicio 3:

Cantaores $\rightarrow x$
Guitarristas $\rightarrow y$
Bailaores $\rightarrow z$

$$\begin{cases} 3x+2y+z=150 \\ x+3y+2z=170 \\ 2x+y+4z=200 \end{cases} \Longrightarrow \left(\begin{array}{ccc|c} \boxed{1} & 3 & 2 & 170 \\ 3 & 2 & 1 & 150 \\ 2 & 1 & 4 & 200 \end{array}\right) \xrightarrow[F_3=F_3-2F_1]{F_2=F_2-3F_1} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 170 \\ 0 & -7 & -5 & -360 \\ 0 & -5 & 0 & -140 \end{array}\right)$$

$$\xrightarrow{F_2=\left(\frac{-1}{7}\right)F_2} \left(\begin{array}{ccc|c} \boxed{1} & 3 & 2 & 170 \\ 0 & \boxed{1} & 5/7 & 360/7 \\ 0 & -5 & 0 & -140 \end{array}\right) \xrightarrow{F_3=F_3+5F_2} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 170 \\ 0 & 1 & 5/7 & 360/7 \\ 0 & 0 & \frac{25}{7} & \frac{820}{7} \end{array}\right) \Longrightarrow$$

$$\Longrightarrow \begin{cases} x+3y+2z=170 \quad (1) \\ y+\frac{5}{7}z=\frac{360}{7} \quad (2) \\ \frac{25}{7}z=\frac{820}{7} \Rightarrow z=\frac{820}{7}:\frac{25}{7} \Rightarrow \boxed{z=32'8€} \end{cases}$$

$$(2)\ y+\frac{164}{7}=\frac{360}{7} \Rightarrow \boxed{y=\frac{196}{7}=28€}$$

$$(1)\ x+3\cdot 28+2\cdot 32'8=170 \Rightarrow \boxed{x=170-65'6-84=20'4€}$$

$\longrightarrow$ Cantaores $(x) \rightarrow 3x+x+2x=6x \rightarrow$ 6 cantaores en total

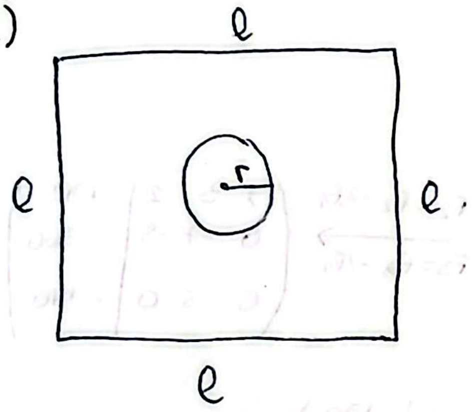
$$\left.\begin{array}{l} 1 \rightarrow 20'4€ \\ 6 \rightarrow x€ \end{array}\right\} x=\frac{6\cdot 20'4}{1} \longrightarrow \boxed{x=122'40€}$$

tuvo que gastarse la peña en cantaores.

$\Longrightarrow$ Cada guitarrista (y) cobró 28€
Cada bailaor (z) cobró 32'80€

Ejercicio 4:

a)



- Área del parque − 2 · Área estanque = 0

  ↳ $l^2 - 2\pi r^2 = 0 \rightarrow l^2 = 2\pi r^2$

- Radio del estanque × lado parque = 400

  ↳ $r \cdot l = 400$

$$\Rightarrow \begin{cases} \boxed{l^2 = 2\pi r^2} \ (2) \\ r \cdot l = 400 \Rightarrow \boxed{l = \frac{400}{r}} \ (1) \end{cases}$$

Sustituyo (1) en (2)

$$\Rightarrow \left(\frac{400}{r}\right)^2 = 2\pi r^2$$

$$\frac{160000}{r^2} = 2\pi r^2$$

$$160000 = 2\pi r^4$$

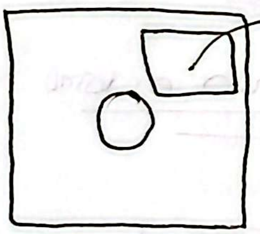
$$r^4 = \frac{160000}{2\pi} \Rightarrow \boxed{r = \sqrt[4]{\frac{80000}{\pi}}}$$

sustituyo en (3)

$$r \cdot l = 400 \Rightarrow \boxed{r = \frac{400}{l}} \ (3)$$

$$\Rightarrow \sqrt[4]{\frac{80000}{\pi}} = \frac{400}{l} \rightarrow \boxed{l = \frac{400}{\sqrt[4]{\frac{80000}{\pi}}} \ m}$$

b)



→ ¿Cabe el minigolf?

Área restante = Área parque − Área fuente.

PRIMERO, SIMPLIFICAMOS:

$$l = \frac{400}{\sqrt[4]{\frac{80000}{\pi}}} = \frac{400}{\frac{\sqrt[4]{80000}}{\sqrt[4]{\pi}}} = \frac{400\sqrt[4]{\pi}}{\sqrt[4]{8 \cdot 10^4}} = \frac{400\sqrt[4]{\pi}}{10\sqrt[4]{8}} = 40 \cdot \sqrt[4]{\frac{\pi}{8}} =$$

$$= 40 \cdot \sqrt[4]{\frac{\pi}{2^3}} = 40 \cdot \sqrt[4]{\frac{2\pi}{2^4}} = \boxed{20\sqrt[4]{2\pi} \ m}$$

$$r = \sqrt[4]{\frac{80000}{\pi}} = \frac{\sqrt[4]{8 \cdot 10^4}}{\sqrt[4]{\pi}} = \frac{10\sqrt[4]{2^3 \cdot 2}}{\sqrt[4]{\pi \cdot 2}} = \boxed{\frac{20}{\sqrt[4]{2\pi}} \ m}$$

Ahora sí:

$$\text{Área parque} - \underbrace{\text{Área estanque}}_{\text{2 veces la del parque.}} =$$

$$\Rightarrow l^2 = 2\pi r^2$$

$$\rightarrow l^2 - 2\pi r^2 = (2\pi r^2) - \pi r^2 = \underline{\underline{\pi r^2}}$$

↳ Área disponible = área estanque

$$\pi r^2 \rightarrow \text{Como } r = \frac{20}{\sqrt[4]{2\pi}}$$

$$\Rightarrow \pi \cdot \left(\frac{20}{\sqrt[4]{2\pi}}\right)^2 = \underbrace{\pi \cdot \frac{400}{\sqrt{2\pi}}}_{} =$$

$$\frac{400\pi}{\sqrt{2\pi}} \cdot \frac{\sqrt{2\pi}}{\sqrt{2\pi}} = \frac{400\cancel{\pi}\sqrt{2\pi}}{2\cancel{\pi}} = \boxed{200\sqrt{2\pi}\ m^2}$$

ÁREA DISPONIBLE

$$201\sqrt{2\pi} > 200\sqrt{2\pi}$$

$\boxed{\text{NO cabe el minigolf}}$

SOLUCIÓN