# Examen Tema 4 Trigonometría

RESOLUCIÓN 2025/26

1.

$$\operatorname{sen}(x) + \operatorname{sen}(2x) + \operatorname{sen}(3x) = 0 \Rightarrow$$

$$\Rightarrow (\operatorname{sen}(3x) + \operatorname{sen}(x)) + \operatorname{sen}(2x) = 0.$$

Aplicamos la fórmula $\operatorname{sen} A + \operatorname{sen} B = 2 \operatorname{sen} \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$

$$\Rightarrow 2 \cdot \operatorname{sen} \frac{3x+x}{2} \cos \frac{3x-x}{2} + \operatorname{sen}(2x) = 0 \Rightarrow$$

$$\Rightarrow 2 \cdot \operatorname{sen}(2x) \cdot \cos(x) + \operatorname{sen}(2x) = 0.$$

Factorizamos $\operatorname{sen}(2x)$:

$$\Rightarrow \operatorname{sen}(2x) \cdot (2\cos x + 1) = 0$$

- Si $\operatorname{sen}(2x) = 0$:

$2x = 0° + 180°K \Rightarrow \boxed{x = 90°K \; ; \; K \in \mathbb{Z}}$

- Si $2\cos(x) + 1 = 0 \Rightarrow \cos x = -1/2$:

$$\boxed{\begin{array}{l} x = 120° + 360°K \\ x = 240° + 360K \end{array}}$$

2.
$$\begin{cases} (x + \operatorname{sen}(x+y))^2 = \operatorname{sen}^2(x+y) + x^2 + 2x \operatorname{sen}(x+y) - \operatorname{sen}(x+y) + 1 & (1) \\ \operatorname{sen} x + \operatorname{sen} y = \frac{\sqrt{6}}{2} & (2) \end{cases}$$

$$(1)\ \cancel{x^2} + \cancel{2x \operatorname{sen}(x+y)} + \cancel{\operatorname{sen}^2(x+y)} = \cancel{\operatorname{sen}^2(x+y)} + \cancel{x^2} + \cancel{2x \cdot \operatorname{sen}(x+y)} - \operatorname{sen}(x+y) + 1 \Rightarrow$$

$$\Rightarrow 0 = -\operatorname{sen}(x+y) + 1 \Rightarrow \operatorname{sen}(x+y) = 1 \Rightarrow x+y = 90° \Rightarrow y = 90° - x.$$

Sustituimos en (2): $\operatorname{sen}(x) + \operatorname{sen}(90-x) = \frac{\sqrt{6}}{2}$ ; $\operatorname{sen}(90-x) = \cos x \Rightarrow$

$$\Rightarrow (\operatorname{sen}(x) + \cos(x))^2 = \left(\frac{\sqrt{6}}{2}\right)^2 \Rightarrow \underbrace{\operatorname{sen}^2 x + \cos^2 x}_{=1} + \underbrace{2 \operatorname{sen} x \cos x}_{= \operatorname{sen}(2x)} = \frac{6}{4} \Rightarrow$$

$$\Rightarrow 1 + \operatorname{sen}(2x) = 1{,}5 \Rightarrow \operatorname{sen}(2x) = \frac{1}{2} \Rightarrow 2x = 30° \Rightarrow \boxed{x = 15°, y = 75°}$$

(y viceversa).



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③ $$\frac{2\cdot \operatorname{sen}(x)\cdot \cos(x)\cdot [\operatorname{sen}^4(x) - \cos^4(x)]}{\operatorname{sen}(x)+\cos(x)} = \operatorname{sen}(2x)\cdot(\operatorname{sen} x - \cos x)$$

1. Sabemos que $2\cdot \operatorname{sen} x\cdot \cos x = \operatorname{sen}(2x)$.

2. Aplicamos diferencia de cuadrados en $[\operatorname{sen}^4(x) - \cos^4(x)]$:

$$\underbrace{(\operatorname{sen}^2(x) + \cos^2 x)}_{=1}\cdot(\operatorname{sen}^2 x - \cos^2 x) = \operatorname{sen}^2 x - \cos^2 x.$$

$\Downarrow$

Aplicamos diferencia de cuadrados:

$(\operatorname{sen} x + \cos x)\cdot(\operatorname{sen} x - \cos x)$

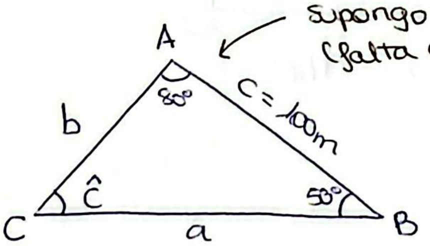
<u>SUSTITUIMOS TODO EN LA FRACCIÓN ORIGINAL:</u>

$$\frac{\operatorname{sen}(2x)\cdot(\cancel{\operatorname{sen} x + \cos x})\cdot(\operatorname{sen} x - \cos x)}{\cancel{\operatorname{sen} x + \cos x}} = \boxed{\operatorname{sen}(2x)\cdot(\operatorname{sen} x - \cos x)}\ \checkmark$$

□ q.e.d.

④

supongo $\hat{A} = 80°$ (falta el dato).




$80 + 50 + \hat{C} = 180°$

$\Rightarrow \boxed{\hat{C} = 50°}$.

$\hat{B} = \hat{C} = 50° \Rightarrow$ ISÓSCELES $\Rightarrow \boxed{b = c = 100m}$

• Para el lado A:

→ <u>TEOREMA DEL SENO</u>:

$$\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} \Rightarrow \frac{a}{\operatorname{sen} 80°} = \frac{100}{\operatorname{sen} 50°} \Rightarrow a = \frac{100\cdot \operatorname{sen} 80°}{\operatorname{sen} 50°} \Rightarrow \boxed{a = 128{,}56m}$$

<u>Resumen de resultados</u>:

→ <u>Ángulos</u>: $\hat{A} = 80°$, $\hat{B} = 50°$, $\hat{C} = 50°$.

→ <u>Lados</u>: $a \approx 128'56m$, $b = 100m = c$.